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About the author

Professor R. Janicki is at the Department of Computer Science and Systems, McMaster University, Hamilton, Ontario, Canada.
Dr. M. Koutny joined the Computing Laboratory in March 1985, where he is a Lecturer

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Structure of Concurrency

Ryszard Janicki
McMaster University, Hamilton, Ontario, Canada L8S 4K1
Maciej Koutny
The University, Newcastle upon Tyne NE1 7RU, U.K.

Abstract

True concurrency semantics assume that behavioural properties of non-sequential systems can be adequately modelled in terms of causal partial orders. We claim that the structure of concurrency is richer, with causality being only one of several invariants that could be used to faithfully characterise concurrent behaviours. The model we are going to investigate has three levels: the Observation level - dealing with single runs or executions; the Invariant level - dealing with invariant relations reflecting the structural properties of the concurrent system; and the System level. Here the presentation is focused on the Observation and Invariant levels and their mutual relationship.

1 Introduction

True concurrency models, e.g. [16], make the assumption that all relevant behavioural properties of concurrent systems can be expressed in terms of causal partial orders. This assumption is arbitrary and the model, although very successful in general, seems to be unable to describe properly some aspects of systems with priorities, inhibitor Petri nets and error recovery systems [5,6,8,11]. We claim that theory of concurrency should also consider other invariants that reflect the structural properties of non-sequential systems, and propose to develop a general approach based upon three levels of abstraction. On the Observation level only single runs, e.g. interleaving and step sequences, are considered. The Invariant level deals with invariant relations on sets of closely related observations, reflecting the structural properties of the underlying system. In particular, this is the level where causal partial orders find their place. The properties of complete systems are dealt with on the System level. In this paper we shall focus on the Observation and Invariant levels and their mutual relationship. The main original aspects are the introduction of different classes of observations, and the notion of a general report system. The proofs of the theorems quoted in this paper can be found in the technical report [6]. For other aspects of the approach, including detailed motivation, the reader is referred to [8,9].

2 Observations

Observation is an abstract model of the execution of a concurrent system. It is a special report supplied by an observer who has to fill in a (possibly infinite) matrix with rows and columns indexed by event occurrences. The observer fills in the entire matrix, except the diagonal, using $\rightarrow$ to denote precedence, $\leftarrow$ following, and $\leftrightarrow$ simultaneity. The transitivity of the precedence relation means that observations can be represented by partially ordered sets of event occurrences, where ordering represents precedence, and incomparability represents simultaneity (see Fig. 1). These partial orders must not be confused with the partial orders of [2,16], where ordering represents causality and incomparability represents independence. (As it was pointed out in [15], causality cannot be observed.) Causal partial orders represent sets of closely related observations and belong to the invariant level. Observations should rather be treated as a generalisation of interleaving and step sequences.

Not all matrices represent partial orders and not all partial orders may be interpreted as valid observations. The additional conditions are that the events be finite, and that the observer can only perceive only a single thread of time, observing finitely many events in a finite period of time.

A partially ordered set (or poset) is a pair $po=(\text{dom}(po),\rightarrow_{po})$ such that $\text{dom}(po)$ is a non-empty set and $\rightarrow_{po}$ is an irreflexive and transitive relation on $\text{dom}(po)$. It is total if for all distinct $a$ and $b$, $a\rightarrow_{po} b$ or $b\rightarrow_{po} a$ holds. It is initially finite if for every $a$ there is only finitely many $b$ such that $a\rightarrow_{po} b$ does not hold. We will denote $a\leftrightarrow_{po} b$ if $a$ and $b$ are distinct incomparable elements of $po$, while $\text{Cuts}_{po}$ will denote the set of maximal antichains, i.e. sets $C$ of incomparable elements such that each $a \notin C$ is comparable with at least one element in $C$. We also define a partial order $C_{po}=(\text{Cuts}_{po},\rightarrow_{po})$, where $B \rightarrow_{po} C$ if $B \neq C$ and there are no $b \in B$ and $c \in C$ such that $c \rightarrow_{po} b$. Posets will often be drawn as Hasse diagrams (see Fig. 2).

The formal definition of posets representing observations of concurrent behaviours can be formulated as follows: An observation, $o \in \text{Obs}$, is an initially finite poset of event occurrences such that $C_{po}$ is total.

Note that the finiteness properties of the observation are guaranteed by the poset's initial finiteness, while the assumption about the single thread of time is captured by total ordering of all the snapshots (maximal antichains). See [6] for more detailed discussion on these.

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Figure 1: Observer report and the corresponding poset.
3 Observations and Interval Orders

We now look closer at the structure of the observation posets. A poset \( po \) is an interval order [3] if \( a \rightarrow_{po} b \) and \( c \rightarrow_{po} d \) implies that \( a \rightarrow_{po} d \) or \( c \rightarrow_{po} b \) holds, i.e. if its graph does not contain a subgraph isomorphic to \( po' \) of Fig. 2. The origin of this notion can be traced back to Wiener’s 1914 paper [17], where interval orders were considered as a means to analyse finite temporal events.

Theorem 1 [3] A countable \( po \) is an interval order iff there are two functions \( \phi, \rho : \text{dom}(po) \rightarrow \mathbb{R} \) such that \( \rho(a) > 0 \) for all \( a \), and \( a \rightarrow_{po} b \iff \phi(a) + \rho(a) < \phi(b) \).

That is, each \( a \) defines an interval \( I(a) = [\phi(a), \phi(a) + \rho(a)] \) such that \( a \rightarrow_{po} b \) iff \( I(a) \) precedes \( I(b) \). Hence simultaneity corresponds to the overlapping of the intervals representing the events. Theorem 1 was strengthened in [6] by adding a condition that \( \phi \) be injective. The properties of interval orders with applications to measurement theory were discussed in [4]. It turns out [6,7] (also implicitly [4]) that being an interval order corresponds to \( C_{po} \) being total.

Theorem 2 \( po \) is an interval order iff \( C_{po} \) is a total order.

This yields an alternative definition of observation posets: Observation is an initially finite interval order of event occurrences. Moreover, Theorem 1 can be improved by taking into account the initial finiteness of observations.

Theorem 3 \( po \) is an observation iff there are \( \phi, \rho : \text{dom}(o) \rightarrow \mathbb{N} \) such that \( \phi \) is injective, \( \rho(a) > 0 \) for all \( a \), and \( a \rightarrow_{po} b \iff \phi(a) + \rho(a) < \phi(b) \).

This essentially means that observations can be constructed using a discrete-time clock. We now shall consider three classes of observations derived under some additional assumptions about the observers and the observed events. The first class of observations is obtained by assuming that the timing of events is uniform, each event taking a unit of time. Thus if \( a \rightarrow_{o} b \rightarrow_{o} c \) then there should be no \( d \) which is simultaneous with both \( a \) and \( c \), since the time gap between the ending of \( a \) and beginning of \( c \) is greater than one. Partial orders with such a property have been studied in the context of measurement theory [4].

\( po \) is a semi-order [12] if \( a \rightarrow_{po} b \rightarrow_{po} c \) implies there is no \( d \) such that \( a \rightarrow_{po} d \) and \( d \rightarrow_{po} c \). Observation \( o \) is uniformly timed, \( o \in \text{Obs}_{u/t} \), if it is a semi-order.

Theorem 4 \( o \) is uniformly timed iff there is \( \phi : \text{dom}(o) \rightarrow \mathbb{R} \) such that \( \phi \) is injective, \( \phi(a) > 0 \) for all \( a \), and \( a \rightarrow_{o} b \iff \phi(a) + 1 < \phi(b) \).

![Figure 2: Relationship between po and C_po.](image-url)
It is worth noting that the $\phi$ in Theorem 4 cannot be integer-valued. Hence the uniformity of event timing will in general mean that observations are constructed using a dense-time clock. The interleaving and step sequences are also included in the model. An interleaving sequence, $p_0 \in \text{Obs}_{\text{ interleaving}}$, is an initially finite total order. A step sequence, $p_0 \in \text{Obs}_{\text{ steps}}$, is an initially finite poset such that if $a \prec b$, $a \prec p_0 b$ and $b \prec p_0 c$ then $a \prec p_0 c$ holds. We thus obtained a hierarchy of different classes of observation: $\text{Obs}_{\text{ interleaving}} \subseteq \text{Obs}_{\text{ steps}} \subseteq \text{Obs}_{\text{ steps}} \subseteq \text{Obs}.$

4 Invariants and Histories

Describing concurrent system solely in terms of the observations it may generate is unsatisfactory for many reasons. In fact most of the arguments made in favour of causal partial orders [2] can also support the introduction of other invariants. We now will focus on the relationship between observations of the same concurrent history, where a concurrent history is essentially an invariant or a set of invariants satisfied by all its observations.

A report set (the first approximation of a concurrent history) is a non-empty set $\Delta$ of observations with the same domain $\text{dom}(\Delta)$. A simple relational invariant of $\Delta$, $I \in \text{SRI}(\Delta)$, is a binary relation on $\text{dom}(\Delta)$ which can be characterised by $(a, b) \in I \iff a \equiv b \land \forall o \in \Delta. \Phi(a, b, o),$ where $\Phi$ is defined by:

$$\Phi := \text{true} \lor a \rightarrow_o b \lor a \leftarrow_o b \lor \neg \Phi \lor \Phi \lor \Phi.$$  

Let $\rightarrow_{\Delta}, \leftarrow_{\Delta}, \leftrightarrow_{\Delta}, \rightarrow_{\Delta}$ and $\leftarrow_{\Delta}$ be simple relational invariants defined respectively by:

$$\Phi_{\rightarrow} = a \rightarrow_o b \quad \Phi_{\leftarrow} = a \leftarrow_o b \quad \Phi_{\leftrightarrow} = a \leftrightarrow_o b.$$  

$$\Phi_{\rightarrow} = a \rightarrow_o b \lor a \leftarrow_o b \lor \Phi_{\rightarrow} = a \rightarrow_o b \lor a \leftrightarrow_o b.$$  

The $\rightarrow_{\Delta}$ and $\leftarrow_{\Delta}$ are called causality, $\rightarrow_{\Delta}$ commutativity, $\leftrightarrow_{\Delta}$ synchronisation, $\rightarrow_{\Delta}$ weak causality. $\rightarrow, \leftarrow, \leftrightarrow, \rightarrow_{\Delta}$ and $\leftarrow_{\Delta}$ will be used to denote mappings which for every report set return respectively $\rightarrow_{\Delta}, \leftarrow_{\Delta}, \leftrightarrow_{\Delta}, \rightarrow_{\Delta}, \leftarrow_{\Delta}$ and $\rightarrow_{\Delta}$ and $\leftarrow_{\Delta}$. We call these mappings invariants, and denote their set by $\text{SRI}$. It turns out that $\text{SRI}(\Delta) = \{\emptyset, \rightarrow_{\Delta}, \leftarrow_{\Delta}, \leftrightarrow_{\Delta}, \rightarrow_{\Delta}, \leftarrow_{\Delta}, \rightarrow_{\Delta}, \leftarrow_{\Delta}, \text{dom}(\Delta) \rightarrow \text{id}_{\text{dom}(\Delta)}\}$, and there is $\Delta$ such that $\text{SRI}(\Delta)$ consists of eight different relations (see Fig. 3).

Due to the symmetry we can in fact consider only four invariants: $\rightarrow_{\Delta}, \leftarrow_{\Delta}, \leftrightarrow_{\Delta}$. Moreover, $\rightarrow_{\Delta}$ and $\leftrightarrow_{\Delta}$ may be expressed by $\rightarrow_{\Delta}$ and $\leftrightarrow_{\Delta}$, so it seems reasonable to ask how to find possibly smallest set of invariants from which the whole $\text{SRI}(\Delta)$ could be derived for a given family of report sets.

A signature of a non-empty family $F$ of report sets is any $S \subseteq \text{SRI}$ such that for all $\Delta, \Delta, \in F$, if $\text{dom}(\Delta) = \text{dom}(\Delta, \in F)$ and $I(\Delta) = I(\Delta, \in F)$ for all $I \in S$, then $I(\Delta) = I(\Delta, \in F)$ for all $I \in \text{SRI}$. $S$ is universal if $F$ is the family of all report sets. $S$ is minimal if no proper subset of $S$ is a signature of $F$ and, furthermore, the following holds: If $J \in S$ and $I \in \text{SRI} - S$ are such that $I(\Delta) \subseteq J(\Delta)$ for all $\Delta$, then $(S - \{J\}) \cup \{I\}$ is not a signature of $F$. It turns out that $\{\rightarrow_{\Delta}, \leftarrow_{\Delta}\}$ and $\{\leftrightarrow_{\Delta}, \rightarrow_{\Delta}\}$ are the only universal minimal signatures.

A history is a report set $\Delta$ which is a complete representation of an ab-
stract computation underlying the observations of $\Delta$ expressed by means of the simple report invariants. That is, by defining suitable invariant relations, we aim at capturing the notion of the abstract computation reflecting the structure of the system.

Let $S \subseteq SRI$. The $S$-closure of a report set $\Delta$, denoted by $\Delta(S)$, comprises observations $o$ with the domain $\text{dom}(\Delta)$ such that for all $I \in S$, if $(a, b) \in I(\Delta)$ then $\Phi_I(a, b, o)$ holds ($\Phi_I$ is defined as above). It turns out that $\Delta \subseteq \Delta(S)$, $\Delta(S) = \Delta(S)(S)$, and if $S$ is a universal signature then $\Delta(S) = \Delta(SRD)$. We now may formulate a

\[
\begin{align*}
\Delta &= \{o_1, o_3, o_8\} \\
\Delta_0 &= \{o_1, \ldots, o_{10}\} \\
S &= \{\equiv, \not\equiv\} \\
\Delta(S) &= \Delta(SRD) = \Delta_0 = \Delta_0(S) = \Delta_0(SRD) \\
\Delta_0 &\in \text{Hist} \\
\Delta &\notin \text{Hist}
\end{align*}
\]

Figure 3: Example showing history, invariant closure, invariants and components (symmetric relationship is represented by undirected arcs.)
central definition of the model.

A history, $\Delta \in \text{Hist}$, is a non-empty report set $\Delta$ such that $\Delta = \Delta(SRI)$.

In other words, history is a report set which can be fully described by the invariants it generates. Moreover, the following essentially describe the same thing: (i) $\Delta$; (ii) $SRI(\Delta)$; (iii) $(R_{\Delta}, \rightarrow_{\Delta})$; (iv) $(\kappa_{\Delta}, \simeq_{\Delta})$; and (v) $(I_1(\Delta), ..., I_k(\Delta))$ where $\{I_1, ..., I_k\}$ is any signature of $F$ such that $\Delta \in F$. In concurrency theory, the causality relation is sometimes treated as an invariant, and sometimes as a set of all observations (step-sequence or interleavings) it generates. Our last remark means that such a dual treatment can be generalised to other invariants in SRI.

5 Report Systems and Their Invariants

Many properties of invariants are independent of the specific representation chosen for observations, and it seems important to separate them from those properties which follow, e.g., from the definition of step sequence. What happens if one extends the notion of observation by adding, e.g., relation representing uncertainty or by using the model similar to Allen structures [1]? What would be the impact of such an extension on the development elsewhere in the model? It turns out that the entire approach can be formulated in fairly general terms of report systems with interval order observations being just a special case.

Let $\Sigma_0$ be a set of objects (such as event occurrences). A relational system $\mu = (\Sigma, r_1, ..., r_k)$ is a report over $\Sigma_0$ if: (i) $\Sigma \subseteq \Sigma_0$ and $r_i \subseteq \Sigma \times \Sigma$ for all $i$; (ii) $r_1 \cup ... \cup r_k = \Sigma \times \Sigma -$id$_\Sigma$; and (iii) $r_i \cap r_j = \emptyset$ for all $i \neq j$. We will use $r_{i,\mu}$ to denote $r_i$. A report system is a non-empty set $RS$ of reports of the same length, i.e. comprising the same number of $r_i$'s.

Note that (ii) expresses the completeness of reports by saying that for all distinct $a, b$ the observer must always report at least one relationship $r_i$, while (iii) captures the consistency of reports by saying that at most one such relationship can be reported.

The interval orders can be treated as special reports with $k = 3$. The report system of concurrent observations $RS_{\text{con}}$ is one comprising reports $(\Sigma, r_1, r_2, r_3)$ such that $\Sigma = \text{dom}(\sigma)$, $r_1 = \rightarrow_o$, $r_2 = \leftarrow_o$ and $r_3 = \leftrightarrow_o$, for some $o \in \text{Obs}$.

The notions of report set, simple relational invariant, signature, universal signature, S-closure, and history (as well as the notions of paradigms, traits and components in the next section), can all be introduced for general report systems. The results obtained for interval orders then become special cases. For more details on this the reader is referred to [6], here we only mention the derived notions of evidence and alibi.

For all $i$, let $R_i(\Delta)$ and $I_i(\Delta)$ be simple relational invariants defined respectively by $\Phi_{R_i} = ar_i, \mu b$ and $\Phi_{I_i} = \neg ar_i, \mu b$. Each $R_i(\Delta)$ is an evidence (it says that something has actually happened), while each $I_i(\Delta)$ is an alibi (it says that
something has definitely not taken place). Note that \( \Delta, \diagdown, \equiv \) are abilis, 
while \( \rightarrow, \leftarrow, \leftrightarrow \) are evidences. The following theorem characterises the role of abilis.

**Theorem 5** \( \{\mathcal{A}_1, \ldots, \mathcal{A}_k\} \) is a universal signature. \( \square \)

### 6 Components and Paradigms

We now look again at the report system of concurrent observations \( RS_{con} \). For each concurrent history \( \Delta \), \( SRI(\Delta) \) can be treated just as any finite family of sets. In particular, one can look at the components \( CSRI(\Delta) = \{\rightarrow, \leftarrow, \leftrightarrow \} \) defined by \( SRI(\Delta) \), as shown in Fig. 4.

A formula stating that a given relationship between \( a \) and \( b \) has been observed is called a *simple trait*. There are three simple traits: \( \psi \rightarrow = \exists \in \Delta. a \rightarrow b \), \( \psi \leftarrow = \exists \in \Delta. a \leftarrow b \) and \( \psi \leftrightarrow = \exists \in \Delta. a \leftrightarrow b \). \( CSRI(\Delta) \) can be defined using the simple traits, e.g., \( \equiv \Delta b \leftrightarrow \psi \rightarrow \wedge \psi \leftarrow \wedge \psi \leftrightarrow \).

Due to the symmetry, we only need to discuss five components: \( \rightarrow, \equiv, \leftrightarrow \) and \( \rightarrow, \leftarrow \). The first component (and an invariant) is *causality*. \( \equiv \) should be interpreted as *concurrency* (two events can be observed simultaneously and in both orders). Both causality and concurrency can be found in true concurrency models. \( \equiv \) represents what is usually referred to as *interleaving* (two events can be observed in both orders, but not simultaneously), and is usually dealt with on the level of observations rather than invariants. The fourth component (and an invariant) can be interpreted as *synchronisation*. It is usually introduced only in its implicit form, e.g., through handshake communication in CCS [14]. \( \rightarrow \) is not to our knowledge part of any of the existing models. We think it captures *disabling* of an event by another event [5].

The approach to concurrency based on causality relation requires that every concurrent history adheres to the following rule: If two events have been observed simultaneously then it is possible to observe them in both orders, and vice versa. We will call this rule a *paradigm* characterising the structure of concurrent histories in the causality based approach. The general paradigms, \( \omega \in \mathcal{P}ar \), are defined by:

\[
\omega := \text{true} | \psi \rightarrow | \psi \leftarrow | \neg \psi | \omega \lor \omega .
\]

The evaluation of the formulas \( \omega \in \mathcal{P}ar \) follows the standard rules (\( \land \) and \( \Rightarrow \) being derived connectives). A history \( \Delta \in \text{Hist} satisfies paradigm \( \omega \), \( \Delta \in \mathcal{P}ar(\omega) \),

![Figure 4: Components of simple relational invariants.](image)
if for all distinct \( a, b \) in the domain of \( \Delta \), \( \omega(a,b,\Delta) \) holds. Two paradigms are equivalent, \( \omega \sim \omega_0 \), if \( \text{Par}(\omega) = \text{Par}(\omega_0) \). \( \sim \) reduces the number of paradigms we need to consider. It turns out that (up to \( \sim \)) there are 32 possible paradigms, each being the conjunction of some of the \( \omega_i \)'s defined by:
\[
\begin{align*}
\omega_1 &= \psi \rightarrow \rightarrow \psi \sim \psi \rightarrow \vee \psi \sim \psi \\
\omega_2 &= \psi \rightarrow \psi \sim \psi \rightarrow \wedge \psi \sim \psi \\
\omega_3 &= \psi \rightarrow \psi \sim \psi \rightarrow \wedge \psi \sim \psi \\
\omega_4 &= \psi \rightarrow \rightarrow \psi \sim \psi \rightarrow \vee \psi \sim \psi \\
\omega_5 &= \psi \rightarrow \psi \sim \psi \rightarrow \wedge \psi \sim \psi \rightarrow \text{false}.
\end{align*}
\]

However, the nature of problems considered in computer science implies that two of the \( \omega_i \)'s may be rejected. The first is paradigm \( \omega_4 \) which excludes sequential composition. The second is \( \omega_5 \) since it excludes systems consisting of completely independent components. We are then left with 8 fundamental paradigms:
\[
\begin{align*}
n_1 &= \text{true} \quad n_2 = \omega_1 \quad n_3 = \omega_2 \quad n_4 = \omega_3 \\
n_5 = \omega_1 \wedge \omega_2 \quad n_6 = \omega_1 \wedge \omega_3 \quad n_7 = \omega_2 \wedge \omega_3 \quad n_8 = \omega_1 \wedge \omega_2 \wedge \omega_3.
\end{align*}
\]

There is a strong relationship between paradigms, components and minimal signatures shown in Table 1. The second column there should be interpreted as an exact characterisation of each of the paradigms, e.g. \( \Delta \in n_6 \leftrightarrow \leftrightarrow \Delta = \rightarrow \Delta = \emptyset \).

Note that \( n_1 \) admits all histories, while the most restrictive paradigm, \( n_8 \), admits \( \Delta \) such that \( \exists \in \Delta \cdot a \leftrightarrow_b \Leftrightarrow (\exists \in \Delta \cdot a \rightarrow_b) \wedge (\exists \in \Delta \cdot b \rightarrow_a) \). Hence \( n_8 \) is the paradigm of true concurrency. Moreover, if paradigm \( n_8 \) holds then \( \rightarrow \Delta \) (causality) is the only invariant that we need, and this fact is a theorem in our approach. We also note that in the most general case, \( n_1 \), the explicit causality invariant is not needed.

Paradigm \( n_3 \) assumes that for every history \( \Delta \), \( (\exists \in \Delta \cdot a \rightarrow_b) \wedge (\exists \in \Delta \cdot a \leftrightarrow_b) \Rightarrow (\exists \in \Delta \cdot a \rightarrow_b) \). It seems to be one of most interesting paradigms. On the one hand, \( n_3 \) is general enough to model adequately priority systems and

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inhibitor nets. On the other hand, when restricted to step sequence observations, its minimal signature \( \rightarrow, \nabla \) can be axiomatised and modelled in terms of relational structures called composets \([8]\). Moreover, the histories corresponding to composets can be modelled by comtraces \([9]\), a significant extension of Mazurkiewicz traces \([13]\).

7 Systems

The development of the system level is still in its initial phase, however, some initial results do already exist. In \([9]\) an invariant semantics for inhibitor nets is defined. The semantics is expressed in terms of composets and comtraces, under the assumption that paradigm \( n_3 \) holds and observations are step sequences.

Bibliographical Comments

This paper is much shortened and refined version of a large part of the technical report \([6]\) with the emphasis on observations and invariants. \([8]\) (and to a lesser extent \([10]\)) provides a detailed motivation, analyses invariants, paradigms and their mutual relationship, but pays little attention to the observation level. \([7]\) presents some initial results on the observations model, while \([9]\) provides an application by developing a formal semantics of inhibitor nets.

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