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The $t/(n-1)$-VP Approach to Fault-Tolerant Software

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Key words — Related software faults, Software fault tolerance, Software reliability, Software safety, System-level fault diagnosis

Acronyms

NMR  \(N\)-modular redundancy
NSCP \(N\) self-checking programming
NVP \(N\)-version programming
NVS Sequential application of NVP
RB Recovery blocks
\(t(n-1)\)-VP \(t(n-1)\)-variant programming

1. INTRODUCTION

The provision of tolerance to anticipated hardware faults has been a common practice for many years and forms a vital part of any dependable computing system. A relatively new development is the fault tolerance techniques for coping with unanticipated faults such as design (typically software) faults [1]. In principle, simple replication of software components is insufficient because software design faults can be reproduced when only redundant copies are made [2]. Software fault tolerance usually requires the application of design diversity. Design diversity is the approach in which two or more variants of a component for redundant computations are independently designed to meet a common service specification. Variants are aimed at delivering the same service, but implemented in different ways in the hope that they do not contain the same design faults. Since at least two variants are involved, tolerance to design faults necessitates an adjudicator [3] (ie, a decision algorithm) that determines a single (assumed to be) error-free result based on the results produced by multiple variants. Several techniques have been proposed for structuring a software system, and providing software fault tolerance: recovery blocks (RB) [2], \(N\)-version programming (NVP) [4], \(N\) self-checking programming (NSCP) [5] and some intermediate or combined techniques (see [6, 7] for example). These techniques should be regarded as complementary to (not as a substitute for) those for achieving software fault avoidance such as verification & validation, software testing, and proof methodology.
The first scheme developed for achieving software fault tolerance was the recovery block scheme [2, 8], in which variants are organized in a manner similar to the standby sparing technique [9] used in hardware. The recovery block approach performs run-time fault detection by augmenting any conventional hardware/software error detection mechanism with an acceptance test applied to the results of execution of one variant. If the test fails, an alternate variant is invoked after backward error recovery is performed. Researchers at UCLA devised another approach, namely the $N$-version programming scheme [4]. The NVP approach is a direct application of the hardware $N$-modular redundancy approach (NMR) [9] to software. $N$ versions (i.e., variants) of a program that have been independently designed are executed in parallel and their results compared by an adjudicator. By incorporating a majority vote, the system can eliminate erroneous results (i.e., the minority) and pass on the (presumed to be correct) results (i.e., the majority). In simple cases the voting can be based on tests for identicality; in general, a more sophisticated and application-oriented test is needed. Note that $N$ variants in this approach may be executed sequentially. Grnarov, Arlat and Avizienis [10] sketched such a sequential application of NVP, called NVS. Laprie et al. [5, 11] in 1987 identified a new scheme: $N$ self-checking programming. The NSCP approach attains fault tolerance by the parallel execution of $N$ self-checking software components. Each self-checking component consists of a pair of variants with a comparator. In particular, one self-checking component is regarded as the active component, and the others are considered as "hot" standby spares.

It must be recognized that the success of a fault tolerance scheme depends to a great extent upon its adjudicator and unreliability in the adjudicator can have a dramatic impact on the overall system reliability [1]. The design for a highly reliable adjudicator generally requires that 1) the adjudication mechanism and variants being checked are as independent as possible, so that they cannot be affected by common faults or related faults; 2) the mechanism itself must be simple enough to guarantee its reliability and the overall system performance. The traditional mechanisms are not entirely satisfactory. In the recovery block software, an acceptance test is used in its adjudication mechanism to provide a last line of detecting errors, but since the test is system-specific, and as such very little specific guidance can be given for its construction, it is
difficult to ensure that the acceptance test and variants will be independent of each other. To overcome this problem, some schemes adopt an adjudication mechanism that selects the results by comparing the outputs of multiple variants. However, a practical adjudicator used in NVP is much more sophisticated than the early simple majority vote, while adjudication mechanisms constructed in NSCP are somewhat too simple to effectively detect the related faults that may occur in the active self-checking components. We develop an alternative in the next section, called \( t/(n-1) \)-Variant Programming \((t/(n-1))\)-VP), which exploits several new research results in the area of system diagnosis [12, 13] for the design of a simplified adjudication mechanism. Our proposed scheme has several favourable characteristics, including 1) the potential ability to tolerate multiple related faults among variants, 2) simple adjudication mechanism that requires only \( O(n) \) result comparison steps, 3) the delivery of correct service even when the number of faulty variants exceeds the bound \( t \) in some fault situations, and 4) possible forms of graceful degradation.

Note also that it cannot be guaranteed that independently designed variants will fail independently (i.e., that faults in the different variants will occur at random and be unrelated) despite the adoption of the design diversity approaches [14, 15]. The dependability analysis of software-fault-tolerant systems must therefore study the effect of related (or dependent) faults. A number of papers devoted to such dependability analysis have appeared in the literature (see [6, 14 - 17] for example). In particular, Arlat, Kanoun, and Laprie [16] developed complete fault classifications and presented a detailed evaluation of NVP and RB. Their analysis concentrated on basic architectures able to tolerate a single fault and thereby the analytical conclusions can hold only for those specific instances. In the third section, we augment published work by analyzing more complex (more general) architectures that tolerate two or more software faults and by carefully identifying the ability of various approaches to tolerate independent and related faults. The results drawn from our analysis provide designers with richer information about the fault tolerance properties of various architectures than the results from traditional analysis, and show evidences that the \( t/(n-1) \)-VP approach is a viable addition or alternative to present schemes for coping with software faults.
Notation

\( A_i \) state of adjudicator's execution
\( B \) state of benign failure caused by an undetected error
\( C \) state of catastrophic failure caused by an undetected error
\( C_i \) result comparator
\( C_X \) probability of catastrophic failure of the \( X \) approach
\( D \) state of detected failure
\( E \) state of software execution
\( F_X \) probability of failure of the \( X \) approach
\( I \) state of software idleness during the specified exposure period
\( N, n \) number of software variants
\( p \) probability that all variants produce the same correct results
\( q_I \) probability of an independent fault in a variant
\( q_A \) probability of an independent fault in the adjudicator
\( q_{AD} \) probability of a detected independent fault in the adjudicator
\( q_{AU} \) probability of an undetected independent fault in the adjudicator
\( q_{AV} \) probability of related faults among the variants and the adjudicator
\( q_{mV} \) probability of related faults among \( m \) variants
\( q_U \) probability of an undetected failure
\( q_C \) probability of a catastrophic failure due to an undetected error
\( R_X(t) \) reliability of the \( X \) approach
\( S_X(t) \) safety of the \( X \) approach
\( U \) state of undetected failure
\( V \) state of variant execution
\( V_i \) \( i \)th software variant
\( \sigma \) departure rate from state \( I \)
\( \omega_{(i, i+1)} \) (comparison) test outcome
Other, standard notation is given in “Information for Readers & Authors” at the rear of each issue. Less frequently used notation may be defined in the text where it first appears.

2. THE $t/(n-1)$-VARIANT PROGRAMMING SCHEME

In the theory of system-level fault diagnosis (see [12, 13] where further references can be found), a particular diagnosability measure, denoted as $t/(n-1)$-diagnosability, was first introduced in [18]. Its diagnosis goal is, for a system composed of $n$ units, to isolate the faulty units to a set of size at most $(n-1)$, under the condition that the number of faulty units is at most $t$. In other words, at least one unit exists such that it is not in the set of size $(n-1)$ and can thus be unambiguously identified as fault-free, provided that the system itself is $t/(n-1)$-fault diagnosable and the number of faulty units in the system does not exceed the bound $t$. Immediately, a natural idea springs to mind: the $t/(n-1)$-diagnosis technique may be employed to select a single correct result from the results generated by $n$ replicated software modules (of independent design). An intuition indicates that we would benefit from the utilization of $t/(n-1)$-diagnosis since this special diagnosis measure cuts down significantly the requirement on the number of tests (i.e., the number of result comparisons) relative to previous diagnosis schemes. It is thus possible to somehow use the idea behind the $t/(n-1)$-diagnosis technique to construct a simple, but dependable adjudication mechanism. Based on current theoretical results of $t/(n-1)$-diagnosis (see [19 - 24] and a subsequent discussion), we develop a new scheme for tolerating hardware and/or software faults. Our description of this scheme is first in terms of application to software fault tolerance, but the approach can also be implemented with hardware [22]. Two classes of software faults are distinguished: independent faults and related faults [11, 16]. Independent faults occur in single variants or in the adjudication mechanism, while related faults can take place among multiple variants and among the adjudicator and one or more variants.

2.1. Description of the $t/(n-1)$-VP Scheme and an Example

In what follows, we shall term such a software fault tolerance scheme $t/(n-1)$-Variant Programming. A general $t/(n-1)$-VP architecture can identify the correct result from
a subset of the results of \( n \) software modules (or variants), provided that the number of faulty modules in the architecture does not exceed \( t \) (i.e., it can tolerate at least \( t \) software faults). The semantics of \( t/(n-1) \)-VP can be expressed more directly as follows:

1) each of \( n \) independently designed software variants is executed in parallel;
2) just some of their results are compared to produce a syndrome;
3) using the syndrome, a diagnosis program performs \( t/(n-1) \)-diagnosis and selects a presumably correct result as the system output (e.g., through switching of the results);

and if no acceptable result is identified, the system will invoke spare software variants, if exists some, or simply signal an exception.

![Diagram](image)

**Figure 1.** A \( t/(n-1) \)-VP architecture with \( n = 5 \) and \( t = 2 \).

For the proposed scheme, we first use a concrete example to demonstrate its ability to tolerate both independent and related faults, and then address its effectiveness for any given \( n \) and \( t \). Figure 1 shows a \( t/(n-1) \)-VP architecture where \( n = 5 \) and \( t = 2 \). This \( 2/(5-1) \)-VP architecture consists of five independently designed software modules, called variants \( V_1, V_2, \ldots, \text{ and } V_5 \), which are executed in parallel in a framework that is intended to cater for up to two simultaneous software faults. Three comparators \( C_1, C_2, \text{ and } C_3 \) are placed at the outputs of variants \( V_1, V_2, V_3, \text{ and } V_4 \) to perform error detection, where \( C_i \) compares the results of \( V_i \) and \( V_{i+1} \) (\( i = 1, 2, 3 \)) and generates the test outcome \( \omega_{i,i+1} \). Three (comparison) test outcomes \( \omega_{12}, \omega_{23}, \text{ and } \omega_{34} \) constitute a syndrome. In particular, the test outcome \( \omega_{ij} = 0 (1) \) if the results of the variants \( V_i \) and \( V_j \) agree (disagree). A diagnosis program, the \( t/(n-1) \)-diagnostor, selects one of the results of \( V_1, V_4, \text{ and } V_5 \) according to the value of the syndrome, and switches service delivery (i.e., the system output) to the selected result. The adjudicator of the architecture
is implemented by the three comparators, the $t/(n-1)$-diagnostor and the output switch. Note
that the result “lines” of $V_2$ and $V_3$ are not connected to the output switch and $V_5$ is not
connected to a comparator. However, we can show below that this architecture is $t/(n-1)$-
diagnosable for $t = 2$; namely, the diagnostor can always select a correct result provided that the
number of (independent or related) faults in variants does not exceed two.

Let $r_1$, $r_2$, ..., and $r_5$ be the results of variants $V_1$, $V_2$, ..., and $V_5$ respectively. Table 1
gives all possible syndromes and the corresponding results that can be unambiguously
diagnosed as correct while assuming that no more than two faults occur simultaneously. For
example, in the case that $\omega_{12} = 0$, $\omega_{23} = 1$ and $\omega_{34} = 0$, a single correct result cannot be simply
identified from among those produced by variants $V_1$, $V_2$, $V_3$, and $V_4$. We can however infer
from the syndrome that two or more of the variants $V_1$, $V_2$, $V_3$ and $V_4$ have generated incorrect
results because one single fault cannot lead to such a syndrome. Hence the result of $V_5$ must be
“correct”. In the case where $\omega_{12} = \omega_{23} = \omega_{34} = 0$, either all of the variants $V_1$, $V_2$, $V_3$ and $V_4$
have to be correct or all of them have to be incorrect. By the previous assumption that $t = 2$,
these results should be classified as acceptable. Following a similar method, we can analyze
other cases to determine the correct results. In fact, table 1 may be viewed as a simple
diagnosis algorithm for the specific architecture. From this table, we find a significant fact that
at least one of results $r_1$, $r_4$ and $r_5$ must be correct for a given syndrome. Accordingly, this
architecture can deliver the correct system output by choosing just among the results of three
variants $V_1$, $V_4$ and $V_5$.

Table 1. Possible syndromes and result selections for the $t(=2)/(n-1=4)$-VP architecture.

<table>
<thead>
<tr>
<th>$\omega_{12}$</th>
<th>$\omega_{23}$</th>
<th>$\omega_{34}$</th>
<th>Presumably Correct Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$r_1$ $r_2$ $r_3$ $r_4$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$r_1$ $r_2$ $r_3$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$r_5$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$r_1$ $r_2$ $r_5$</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$r_2$ $r_3$ $r_4$</td>
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<td>1</td>
<td>$r_5$</td>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$r_3$ $r_4$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$r_5$</td>
</tr>
</tbody>
</table>
Unlike the NVP scheme and its variations, $t/(n-1)$-VP does not have to make pairwise comparisons among the results of $n$ variants in order to identify a presumably correct result. It is however interesting to study how many result comparisons (corresponding to the comparators illustrated in Figure 1) are normally required for a general $t/(n-1)$-architecture. In the simplest case that $n = 3$ and $t = 1$, one comparator is necessary and sufficient for $t/(n-1)$-diagnosis — the third result must be acceptable when the two compared results disagree; otherwise they can be identified as correct. Larger $n$ and $t$ require a more deliberate comparison assignment among the results of multiple variants so as to guarantee $t/(n-1)$-diagnosability. For example, result comparisons of $n$ variants could be organized into a form of chains, where the comparator $C_i$ $(1 \leq i \leq n-1)$ compares the results of variants $V_i$ and $V_{i+1}$. Alternatively, result comparisons may be organized into a more complex structure, $H_{2r,n}$, where the result of variant $V_i$ $(1 \leq i \leq n)$ is compared with that of $V_j$ if and only if $i-r \leq j \leq i+r \ (mod \ n+1, \ r = 1, 2, 3, ...\)$. Theorem 4 in [22] gave several sufficient conditions on the result comparison assignments of the systems that are $t/(n-1)$-diagnosable, and it is reproduced here in order to make this paper self-contained.

**Theorem:** A system $S$ composed of $n$ units (or software variants) is $t/(n-1)$-diagnosable if $n \geq 2t + 1$ and the assignment of result comparisons in the system $S$ contains at least

1) a chain of $t + 1$ units for $t = 1$;

2) a chain of $t + 2$ units for $t = 2$;

3) a chain of $2t + 1$ units for $3 \leq t < 5$;

4) an $H_{2r,n}$ structure with $r = 1$ for $5 \leq t < 7$;

5) an $H_{2r,n}$ structure with $r \geq (t-1)/5$ for $7 \leq t$.

Because the major aim of this paper is to show how the $t/(n-1)$-diagnosis technique could be applied to the design of software fault tolerance schemes, we will not further discuss this particular technique itself (the interested reader is referred to [20, 22 - 24] for more technical details). The diagnosis algorithms with respect to the testing assignments in the above theorem have been developed in [23] for chains and in [24] for $H_{2r,n}$-type systems. For practical values of $n$ (eg, $3 \leq n \leq 10$), a $t/(n-1)$-VP architecture uses only $O(n)$ comparators and
contains a simple diagnosis algorithm with linear complexity. The adjudicator in such an architecture would be simpler than a voter used in NVP (which has to be based on \( O(n^2) \) result comparison steps).

It is important to realize that for any fault tolerance scheme the correctness of results output by the system cannot always be guaranteed (e.g., when more than \( t \) faults have occurred), and moreover such fault situations cannot be detected completely. However, this does not present severe problems; there are acceptable probabilities of catastrophic events in practice (e.g., an aircraft computer system usually accepts the probability of failure that is less than \( 10^{-9} \) per hour in a ten hour flight [25]). Dependability studies that have been performed for practical fault-tolerant systems can be used to determine the probability of the occurrence of \( t \) faults. This helps to make an appropriate design decision as to which scheme is likely to be most effective and how many variants are sufficient for a particular application. Additional fault detection and exception-handling techniques [26] can also be used to improve fault coverage and fault tolerance. In the \( 2/(5-1) \)-architecture of Figure 1, for example, exception-handlers can be incorporated into the variants. The function of the handlers in a variant is to handle any errors that are detected during the execution of the variant, signalling an exception to the diagnostor. The diagnostor comes to final decision according to the value of the syndrome and the exception signals received so far: either it delivers a presumably correct result, or signals a failure exception.

2.2. Comparison with Other Schemes

The \( t/(n-1) \)-VP scheme has resemblance with other fault tolerance techniques that have been previously proposed and examined, especially with those requiring the use of result comparisons such as NVP and NSCP. The fact that the variants are executed in parallel necessitates an input consistency mechanism and a synchronization regime, based essentially on wait and send primitives, and incorporating a time-out mechanism. However, in each case there are significant and fundamental distinctions. Correct results in \( t/(n-1) \)-VP are not obtained by majority vote (as in NVP), or by detecting and discarding erroneous results (as in NSCP), but by \( t/(n-1) \)-diagnosis.
It could be argued that \( t/(n-1) \)-VP is only a variation of NVP; however, in our opinion, the majority voting check is an integral part of NVP, and each of \( N \) software versions in NVP is of equal importance. In marked contrast, the \( t/(n-1) \)-VP scheme does not try to find a majority of \( n \) results, but just to identify a presumably correct result. It can therefore deliver correct results with some probability even when the majority of results of \( n \) variants are incorrect. Moreover, \( t/(n-1) \)-VP has more flexible architectural features. In the architecture of Figure 1, the variant \( V_1 \) can be considered as being active, actually delivering the system output in the absence of faults; the variant \( V_4 \) and \( V_5 \) are used as "hot" spares, and \( V_2 \) and \( V_4 \) are only exploited for detecting errors and producing test outcomes. In addition, NVP requires that all variants should be designed to produce the results that are essentially identical. This constraint can be loosened in the \( t/(n-1) \)-VP approach. While the primary variant \( V_1 \) in the \( 1/(5-1) \)-VP architecture should attempt to produce the desired output, the spare variant \( V_5 \) may only attempt to provide a degraded service. In this form, the \( t/(n-1) \)-VP architecture can be used to implement a type of graceful degradation.

In principle, \( t/(n-1) \)-VP is also different from NVS (a form of sequential NVP) [10]. The \( t/(n-1) \)-VP method is based on so-called hot-standby redundancy, whereas NVS utilizes the cold-standby technique. More precisely, in the case that the results of the first two variants disagree (assuming \( N = n = 3 \), \( t/(n-1) \)-VP will select the result of the third variant, which has been available, as the system output through the result switch. NVS however has to first execute the third variant on the same set of input values and then makes a further decision by searching for a majority of the results. This validation process requires the extra execution time for the third variant and for the final decision. Clearly, in comparison with our scheme, NVS has relatively poor predictability of task completion time and may be inappropriate for certain time-critical applications.

It could be argued that \( t/(n-1) \)-VP is somewhat similar to NSCP. However, a fundamental distinction between the two schemes concerns their capacity for tolerating related faults. NSCP will fail (and even cause catastrophic consequences) whenever the two variants that form the active self-checking component produce the identical, but incorrect results (no
matter how many spares are still available). In contrast, the $t/(n-1)$-VP scheme can tolerate up to $t$ (independent or related) faults; that is, it can deliver the correct service even if $t$ faulty variants compute identical incorrect results.

Finally, from the previous overview of software fault tolerance schemes, it is evident that the $t/(n-1)$-VP approach is quite distinct from the recovery block concept [2]. Like NVP and its variations, $t/(n-1)$-VP is complementary in many respects to RB. Recovery blocks can be more appropriate for those systems where hardware resources are limited and comparison-based adjudicators are inappropriate (a very detailed discussion of the relative advantages and disadvantages of NVP and RB is given in [1]). In the interests of simplicity and brevity, we will focus on the comparison of $t/(n-1)$-VP with NVP and NSCP without further discussing the recovery block approach.

3. DEPENDABILITY EVALUATION OF SOFTWARE FAULT TOLERANCE APPROACHES

In this section, we shall conduct a dependability evaluation of the $t/(n-1)$-VP scheme and the other similar approaches. Arlat, Kanoun and Laprie [16] recently analysed some special architectures using the RB, NVP and NSCP schemes respectively (mainly providing software redundancy able to tolerate single software faults). We exploit their modeling framework for investigating the software redundancy needed to tolerate two or more faults and establish a slightly different model to show the different impacts of independent and related faults on software dependability. Three architectures are analysed that can tolerate at least two software faults: $t/(n-1)$-VP and NVP using five variants, the former adopting a simple diagnosis algorithm for result selection (see table 1) and the latter employing the usual majority adjudication, and NSCP using six variants organized as three self-checking components. (Note that the NSCP architecture being considered here can tolerate two faults in most fault situations except the related faults that occur in an active self-checking component.) Expressions for $F_X$ and $C_X$, where $X \in \{t/(n-1)$-VP, NVP, NSCP\}, will be derived using a Markov approach.
3.1. Underlying Assumptions

1) During the execution of the $X$ scheme, related faults manifest themselves in the form of similar errors, whereas independent faults only cause distinct errors; and furthermore similar errors lead to common-mode failures, and distinct errors only cause independent failures;

2) all variants have the same probability of fault manifestation (or error);

3) only a single fault type, either independent or related, may appear during the execution of the scheme and no compensation [28] may occur between errors of the variants and of the adjudicator, ie, either an error is detected or it causes an incorrect output;

4) probabilities of independent and related faults are significantly low such that the probability $p$ can be approximated to 1 (as assumed by others in similar settings; see [16] for example).

It is worth notice that these assumptions are used only to simplify the notation and the complexity in modeling and should not alter the significance of analytical conclusions to be derived. In particular, assumption 2) can be easily generalized to the case where the variants have respective fault characteristics. More complex models can be developed without applying assumption 4), ie, probabilities of independent and related faults are allowed to be arbitrary (descriptions about these models are provided in [17, 27]).

3.2. Detailed Reliability and Safety Models

We consider in this paper two different but complementary attributes of dependability: the continuity of service and the non-occurrence of catastrophic failure [28]. In general, we define software reliability as a measure of the time to failure and its safety as a measure of the time to catastrophic failure [16, 28]. The time (or the specified exposure period) in this definition is a relative concept and may mean a single run, a number of runs, or time expressed in calendar or execution time units of software. In the case of multiple runs, software may be idle between its executions. However software faults can manifest themselves only when
software is executed. We will therefore focus on the execution process of software. Figure 2 shows a slight variation of the software behaviour model proposed by Arlat et al. [16]. In this behaviour model, a detected failure (i.e., no service is delivered) is classified as *benign*; an undetected failure (i.e., an incorrect result is delivered) can be either benign, or *catastrophic*. Since several runs are possible, service delivery may be restored from benign failures. Note that transitions from $D$ and $B$ to $I$ and from $U$ to $B$ or $C$ are applied only to the safety evaluation. Based on a Markov approach to modeling, reliability of the $X$ approach can be evaluated simply by:

$$R_X(t) = e^{-(\sigma F_X)t}$$

where $\sigma$ is the departure rate from state $I$, $F_X$ the probability of failure of the $X$ approach, and $t$ the specified exposure time (for a detailed discussion of this formula see [16]);

and safety by:

$$S_X(t) = e^{-(\sigma C_X)t}$$

where $\sigma$ is the departure rate from state $I$, $C_X$ the probability of catastrophic failure of the $X$ approach, and $t$ the specified exposure time.

![Figure 2. A modified behaviour model.](image)

3.3. The $t/(n-1)$-VP Model for the 2/(5–1)-Architecture

Figure 3 describes a state-transition diagram for the 2/(5–1)-architecture based on the notation introduced at the end of the Introduction. Respectively, execution states of the adjudicator are explained as follows.

1) State $A1$ corresponds to the case in which five variants produce the same correct results. According to assumption 4), probability $p$ that all variants produce
correct results can be approximated to $1 - 5q_I - 10(q_I)^2 - 10(q_I)^3 - 5(q_I)^4 - (q_I)^5 - 10q_{2}\mathcal{V} - 10q_{3}\mathcal{V} - 5q_{4}\mathcal{V} - 5q_{5}\mathcal{V} - q_{A\mathcal{V}1} (\approx 1)$. Given no fault in any variant, different types of adjudicator failure will lead to states $D$ and $U$ with respective probabilities $q_{AD}$ and $q_{AU}$.

2) States $A2$ and $A3$ indicate activation of one or two independent faults in variants given no related fault among the variants. These fault types can be tolerated by this $2/(5-1)$-architecture.

3) States $A4$, $A5$ and $A6$ correspond to cases in which three or more independent faults manifest themselves in variants. Since the number of faults has exceeded the bound 2, these states may lead to a failure state. However, through a more precise analysis, it is found that $t/(n-1)$-VP can still deliver a correct result in some situations (see a further discussion below).

4) State $A7$ represents activation of related faults in any two variants. These faults can be tolerated.

5) States $A8$, $A9$ and $A10$ correspond to cases in which related faults manifest themselves in more than two variants, which are undetectable.

![Figure 3. The $t/(n-1)$-VP model.](image)
6) State A11 corresponds to activation of related faults between the adjudicator and the variants. This is also regarded as undetectable (see assumption 3).

In this 2/(5-1)-VP model, there is the transition from state A4 (or A5) to state I, that is, the architecture considered may still select a correct result as the system output even in the presence of more than two faults. Without loss of generality, take state A4 as an example. If three independent faults affect only three of variants V_1, V_2, V_3, and V_4, by assumption 1) their results will generate the syndrome where \( \omega_{12} = \omega_{23} = \omega_{34} = 1 \). The result of V_5 (a correct result) will then be chosen as the system output. Note that this class of events may occur with the probability 4q_T^3. Similarly, if three independent faults affect only V_1, V_2 and V_5 (or only V_3, V_4 and V_5), according to table 1, the selected result can be still a correct one, with the probability 2q_T^3. To sum up, the conditional probability of the transition from state A4 to state I is \( (4q_T^3 + 2q_T^3) / (10q_T^3) = 0.6 \). Therefore, the transition from A4 to a failure state can actually take place with the conditional probability \( (4q_T^3) / (10q_T^3) = 0.4 \).

From the state-transition diagram, it follows that
\[
F_{ul(n-1)-VP} = p(q_{AD} + q_{AU}) + 4(q_T)^3 + 4(q_T)^4 + (q_T)^5 + 10q_3V + 5q_4V + q_5V + q_{AV1}
\]

A close but pessimistic approximation can be:
\[
F_{ul(n-1)-VP} = q_{AD} + q_{AU} + 4(q_T)^3 + 4(q_T)^4 + (q_T)^5 + 10q_3V + 5q_4V + q_5V + q_{AV1} \quad (1)
\]

For evaluation of safety, only state C is absorbing; we thus have:
\[
C_{ul(n-1)-VP} = q_C[q_{AU} + 4(q_T)^3 + 4(q_T)^4 + (q_T)^5 + 10q_3V + 5q_4V + q_5V + q_{AV1}] \quad (2)
\]

3.4. The NVP Model for the 5VP-Architecture

The NVP model for the 5VP-architecture is shown in Figure 4. The detailed analysis is essentially similar to that made for \( ul(n-1)-VP \). A major difference is the case where multiple independent faults have an impact on three or more variants. In NVP, this case is much simpler — these faults will always lead to state D, assuming they are always detectable (but not tolerated). Thus, for reliability, \( F_{NVP} \) will be greater than \( F_{ul(n-1)-VP} \):
\[
F_{NVP} = q_{AD} + q_{AU} + 10(q_T)^3 + 5(q_T)^4 + (q_T)^5 + 10q_3V + 5q_4V + q_5V + q_{AV2} \quad (3)
\]

However, due to the detectability of multiple independent faults we have for safety:
\[ C_{\text{NVP}} = q_C[q_{\text{AU}} + 10q_{3V} + 5q_{4V} + q_{5V} + q_{AV_2}] \] (4)

which is obviously lower than \( C_{t(n-1)-VP} \).

3.5. The NSCP Model for the 3SCP-Architecture

Figure 5 shows the NSCP model for the 3SCP-architecture. The interpretations of the states are similar to those of the \( t(n-1) \)-VP model though there are thirteen states \( A_1 \sim A_{13} \) to consider because of the use of six variants. Independent faults in one or two of variants can be tolerated. Independent faults in three or more variants can be either tolerated or detected, as indicated by states \( A_4 \) and \( A_5 \). This shows that the NSCP scheme is quite effective on the treatment of independent faults.
However, cases where related faults manifest themselves among multiple variants become more complicated. On one hand, NSCP is not fault-tolerant in the worst case — any related faults in active self-checking components could lead to certain failure states. On the other hand, some related faults can be tolerated or detected if they do not affect the pair of variants in an active self-checking component. Consider a representative case, state A9, in which related faults manifest themselves in three of the six software variants. There are three sub-cases to consider.

1) If related faults only occur in the spare self-checking components or such faults affect just a variant in the active component but not affect the first spare component, the 3SCP architecture can select a correct result and provide normal service. The conditional probability that this sub-case occurs is \( [(6/20) \times (20q_{3V})] / (20q_{3V}) = 0.3 \).

2) If related faults affect exactly one variant in every self-checking component, they can be detected effectively; the corresponding conditional probability is \( [(2^3/20) \times (20q_{3V})] / (20q_{3V}) = 0.4 \).

3) The worst sub-case is that related faults have an influence upon the pair of variants in the active component or an impact on the pair of variants in the first spare component given these related faults have affected a variant in the active one. In this sub-case, the 3SCP architecture will produce incorrect outputs, and the corresponding conditional probability is \( [(6/20) \times (20q_{3V})] / (20q_{3V}) = 0.3 \).

A similar analysis can be applied to other states. It therefore follows from the state-transition diagram:

\[
F_{NSCP} = q_{AD} + q_{AU} + 8(q_i)^3 + 12(q_i)^4 + 6(q_i)^5 + (q_i)^6 + q_{2V} + 14q_{3V} + 14q_{4V} + 6q_{5V} + q_{6V} + q_{AV3}
\]

(5)

Since independent faults can be either tolerated or detected, safety of the NSCP architecture concerns only related faults:

\[
C_{NSCP} = q_{C}[q_{AV} + q_{2V} + 6q_{3V} + 14q_{4V} + 6q_{5V} + q_{6V} + q_{AV3}]
\]

(6)
3.6. Remarks

From table 2 that summarizes the specific expressions for $q_I$'s and $q_U$'s, it is found that independent failures of the variants have a relatively small influence upon $t/(n-1)$-VP, but a larger impact on NVP and even more on NSCP. This is because the $t/(n-1)$-VP scheme possesses one of the significant characteristics of the $t/(n-1)$-diagnosis technique; namely, it is still possible in some fault situations for our proposed scheme to identify the correct results though faulty variants are in the majority.

Table 2. Specific expressions for $q_I$'s and $q_U$'s.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$t/(n-1)$-VP</th>
<th>NVP</th>
<th>NSCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{I,X}$</td>
<td>$4(q_I)^3 + 4(q_I)^4 + (q_I)^5$</td>
<td>$10(q_I)^3 + 5(q_I)^4 + (q_I)^5$</td>
<td>$8(q_I)^3 + 12(q_I)^4 + 6(q_I)^5 + (q_I)^6$</td>
</tr>
<tr>
<td>$q_{A,X}$</td>
<td>$q_{A1}$ (comparators)</td>
<td>$q_{A2}$ (voter)</td>
<td>$q_{A3}$ (comparator)</td>
</tr>
<tr>
<td>$q_{AV,X}$</td>
<td>$q_{AV}$</td>
<td>$q_{AV2}$</td>
<td>$q_{AV3}$</td>
</tr>
<tr>
<td>$q_{mV,X}$</td>
<td>$10q_3v + 5q_4v + q_5v$</td>
<td>$10q_3v + 5q_4v + q_5v$</td>
<td>$q_2v + 6q_3v + 14q_4v + 6q_5v + q_6v$</td>
</tr>
<tr>
<td>$q_{U,X}$</td>
<td>$q_{AU} + 4(q_I)^3 + 4(q_I)^4 + (q_I)^5 + 10q_3v + 5q_4v + q_5v + q_{AV}$</td>
<td>$q_{AU} + 10q_3v + 5q_4v + q_5v + q_{AV2}$</td>
<td>$q_{AU} + q_{2v} + 6q_3v + 14q_4v + 6q_5v + 96v + q_{AV3}$</td>
</tr>
</tbody>
</table>

As assumed previously, the adjudicators used in the three specific architectures are: result comparison plus a diagnosis algorithm in $t/(n-1)$-VP, a voter in NVP, and result comparison (plus the result switch) in NSCP. According to their complexities, it would be reasonable to rank $q_A$'s and $q_{AV}$'s as follows.

$$q_A(\text{NSCP}) \leq q_A(\text{$t/(n-1)$-VP}) \leq q_A(\text{NVP})$$  \hspace{1cm} (7)

$$q_{AV}(\text{NSCP}) \leq q_{AV}(\text{$t/(n-1)$-VP}) \leq q_{AV}(\text{NVP})$$  \hspace{1cm} (8)

It follows from the table that related faults among variants have the same influence upon $t/(n-1)$-VP and NVP, but more serious on NSCP. This is a consequence of the fact that result comparison used in the self-checking components and the NSCP architecture itself are not effective enough to detect (or further tolerate) the related faults that may affect both variants in a self-checking component. Generally, this cannot be overcome by incorporating more variants.
into a given architecture. In contrast, both \( t/(n-1) \)-VP and NVP can tolerate certain number of related faults under the same bound and furthermore their fault tolerance capability can be enhanced, at least in principle, by involving more software variants.

Summarizing, the analysis above could thus suggest the following general conclusions.

For reliability:

\[
F_{t/(n-1)-VP} < F_{NVP} < F_{NSCP}
\]  

(9)

The inequality (9) means that the \( t/(n-1) \)-VP architecture has the lowest probability of failure — equivalently, the highest reliability. Due to high detectability of independent faults, NVP is however less sensitive to undetected faults than \( t/(n-1) \)-VP. The probability \( q_{U(t/(n-1)-VP)} \) for \( t/(n-1) \)-VP looks relatively high since this scheme may fail to detect some independent faults when the bound on the number of faulty variants is violated. This probability could be reduced by using more software variants. Note that the probability \( q_{U(NSCP)} \) is high as well, but again the incorporation of more variants would be of no effect on safety enhancement of NSCP. So for safety:

\[
C_{NVP} < C_{t/(n-1)-VP} \leq C_{NSCP}
\]  

(10)

Finally, it is important to notice that the evaluation data obtained here are used only to uncover the relative advantages and disadvantages of these schemes under consideration. For a given design using a particular scheme, the evaluation results also show how the design could be modified to further improve its dependability. Since the notion of software dependability captures many different concerns, including the qualities of reliability and safety, our analysis demonstrates the need of a delicate balance between these complementary attributes. In practice, a software designer must make an objective decision as to which technique is likely to be most appropriate for a specific application.
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